جـامـعـة بنهـــا

كلية الحاسبات والذكاء الإصطناعي

Calculus

Lecture 01

Dr. Ahmed Hagag

Faculty of Computers and Artificial Intelligence
Benha University

Spring 2023

Introduce Myself

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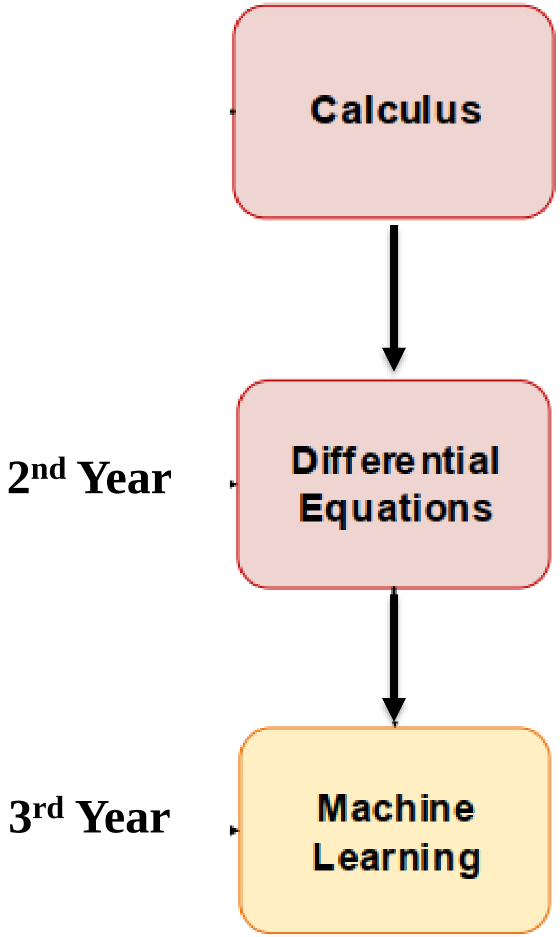
Basic Course Information

- Course code: **BS101**
- Course name: **Calculus**
- Level: **1st Year / B.Sc.**
- Course Credit: **3 credits**
- Instructor:

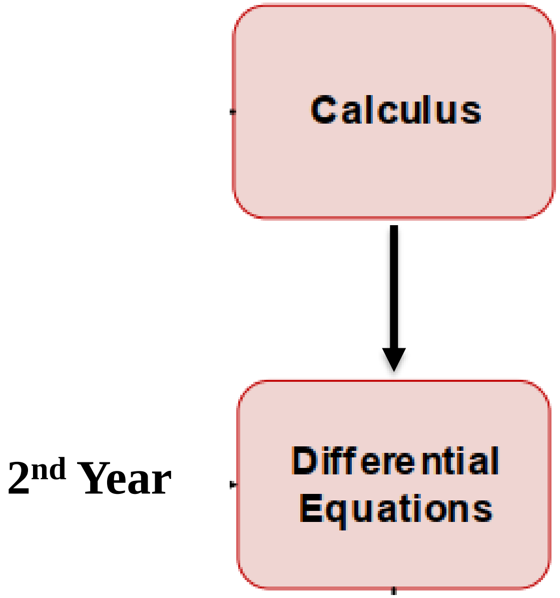
Dr. Mustafa Hassan

Dr. Ahmed Hagag

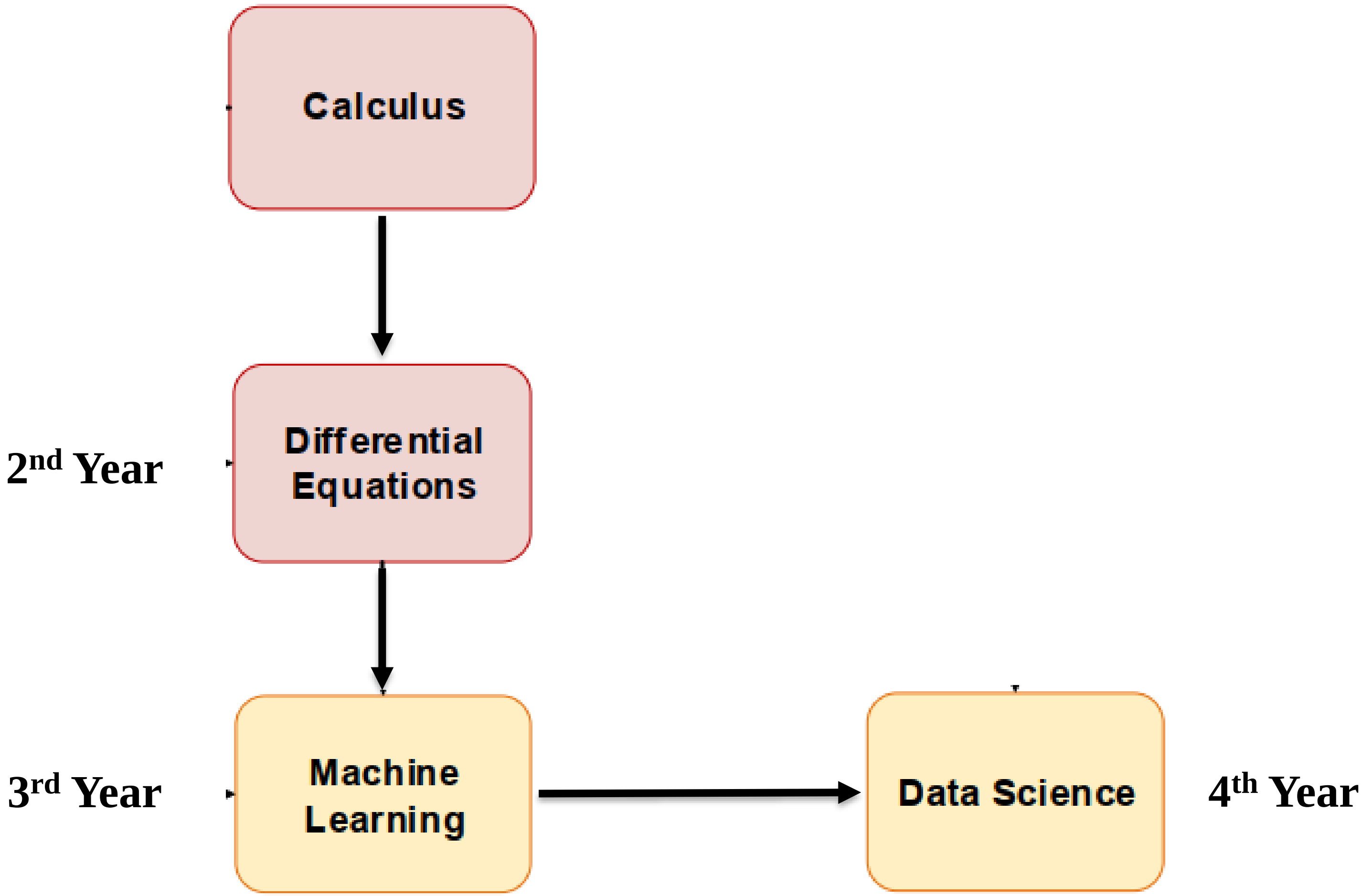
Flowchart (CS)



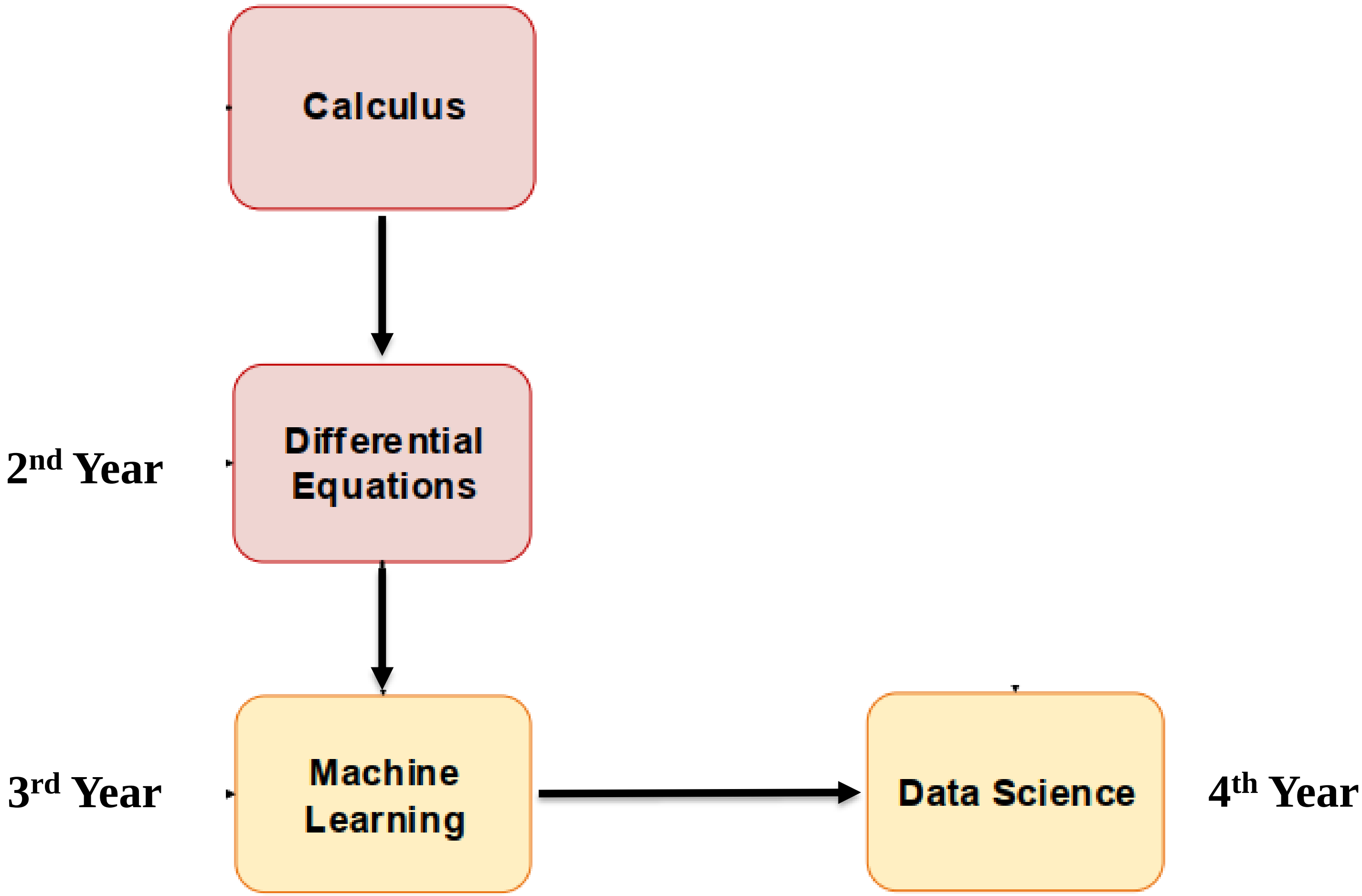
Flowchart (IS)



Flowchart (SC)



Flowchart (AI)



Assessment (1/6)

Final Exam

50

الامتحان النهائي

Section

15

حضور و واجبات
ومشاركة في السكاشن

Midterm

20

منتصف الفصل

Oral

10

الشفوي

Attend

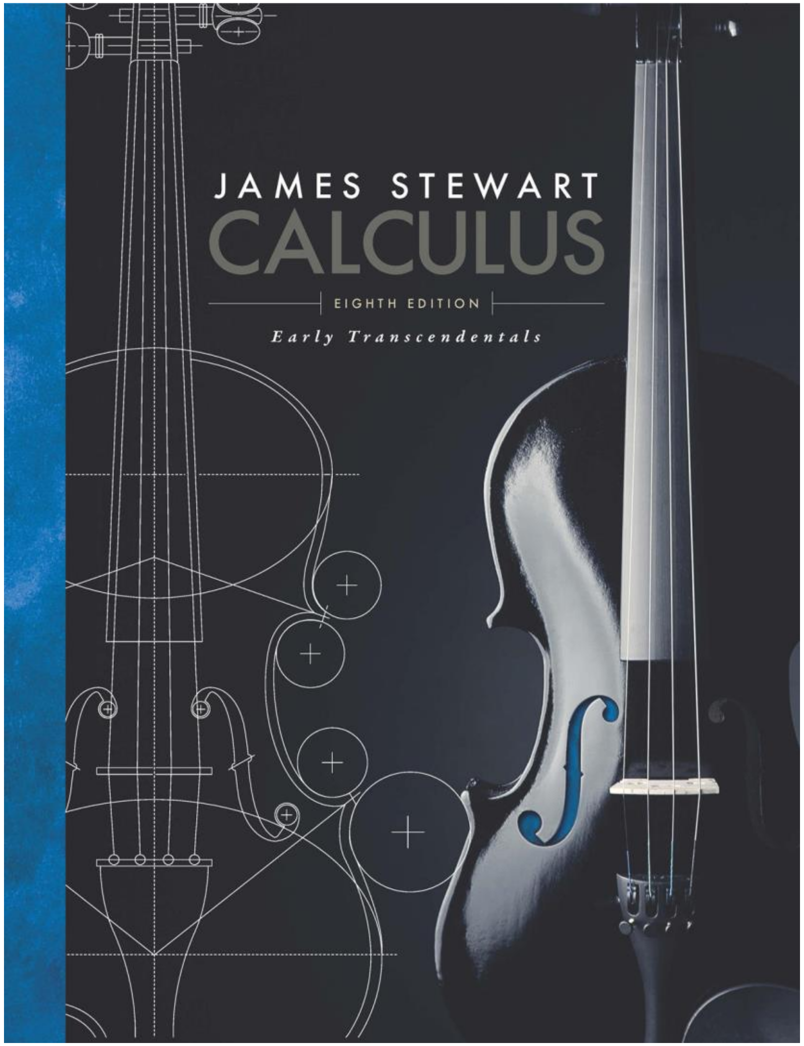
5

حضور

General Discussion

What do you expect to get from this course?

Lectures References (1/3)

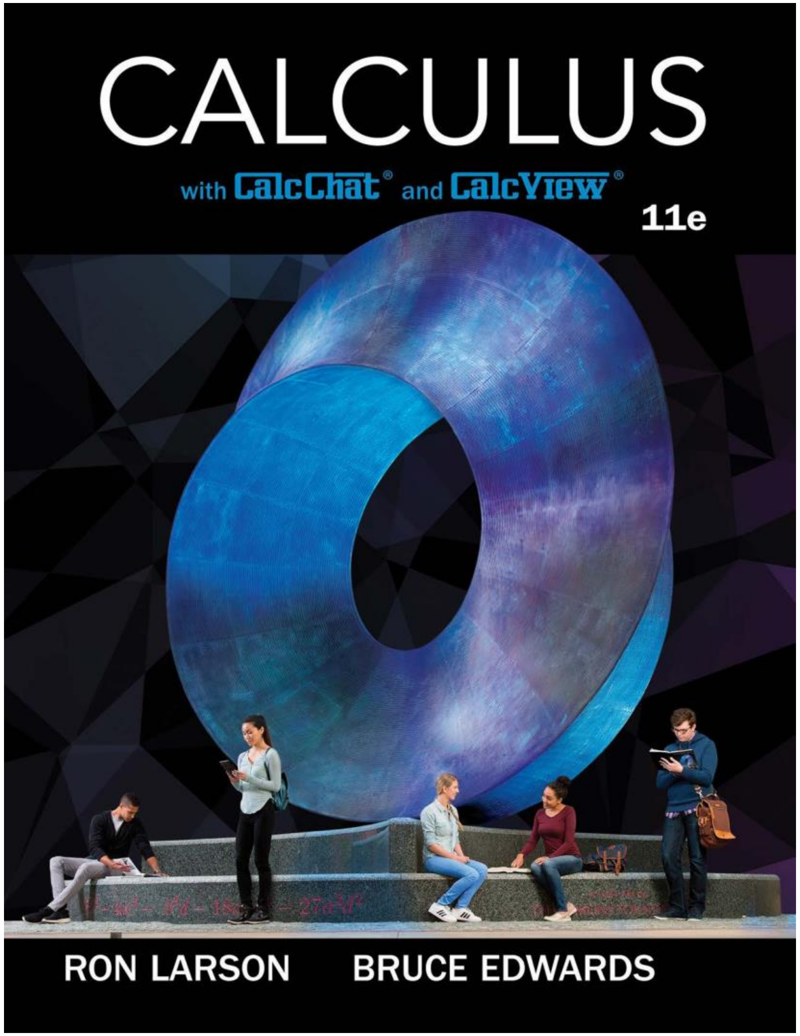


Calculus

James Stewart

8 Edition

Lectures References (2/3)

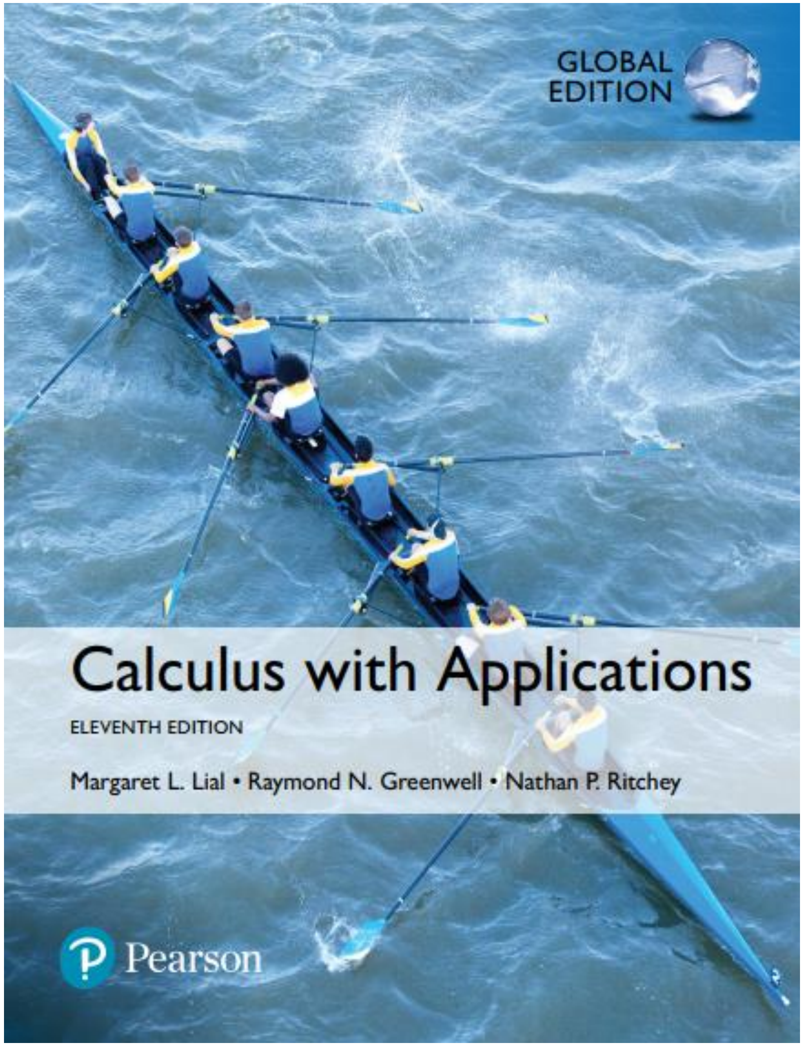


Calculus

Ron Larson and Bruce Edwards

11 Edition

Lectures References (3/3)



Calculus with Applications

Margaret Lial, Raymond Greenwell and Nathan Ritchey

11 Edition

Calculus

Differentiation & Integration

1665 – 1675

Isaac Newton
(England)

Gottfried Leibniz
(German)

Related Topics

Calculus

Differentiation & Integration 1665 – 1675

Limits 1821

Augustin-Louis Cauchy
(France)

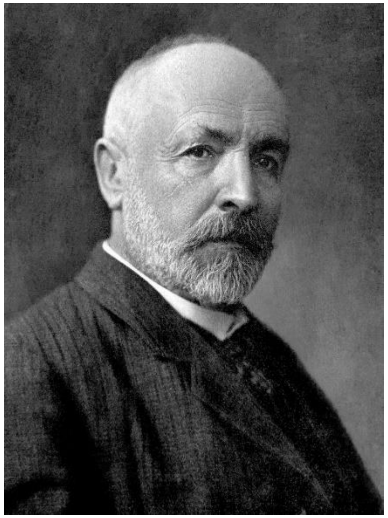
Related Topics

Calculus

Differentiation & Integration **1665 – 1675**

Limits **1821**

Set Theory **1874**



Georg Cantor
(German)

Related Topics

Calculus

Differentiation & Integration — **1665 – 1675**

Limits — **1821**

Set Theory — **1874**

Function — **From the 12th Century. Concept function in 1673**

Gottfried Leibniz
(German)

Related Topics

Calculus

Differentiation & Integration	1675
Limits	1821
Set Theory	1874
Function	1673
Numbers	

Related Topics

Calculus

Topic	Group	Year
Differentiation & Integration	3	1675
Limits	2	1821
Set Theory	1	1874
Function	1	1673
Numbers	1	

Course Syllabus

- Chapter 1: Numbers, Sets, and Functions.
- Chapter 2: Limits and Continuity.
- Chapter 3: Derivatives and Differentiation Rules.
- Chapter 4: Applications of Differentiation.
- Chapter 5: Integrals.
- Chapter 6: Techniques of Integration.
- Chapter 7: Applications of Definite Integrals.

Chapter 1 Topics

- Numbers and Sets.
- Representations of Functions.
- Domain & Range of Functions.
- Algebra of Functions.
- Increasing and Decreasing.
- Test for Even and Odd Functions.
- Types of Functions and their Graphs.
- Transformations of Functions.

Numbers (1/5)

The simplest numbers are the "counting numbers"

$$1, 2, 3, \ldots$$

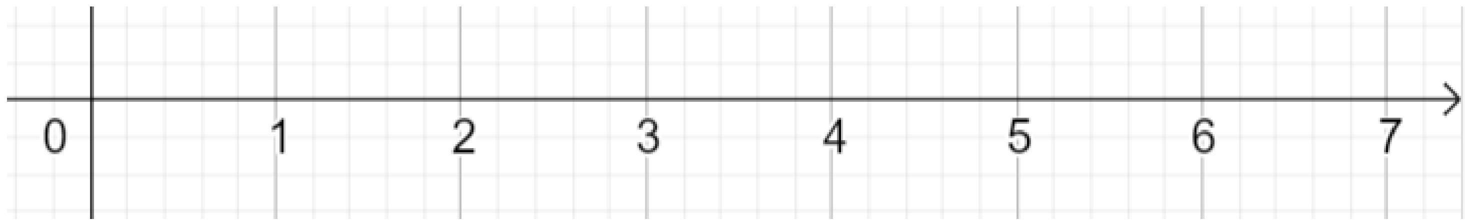
The fundamental significance of this collection of numbers is emphasized by its symbol **N** (for **natural numbers**). Also, begin the natural numbers with 0, to be

$$0, 1, 2, 3, \ldots$$

Also called the **whole numbers**.

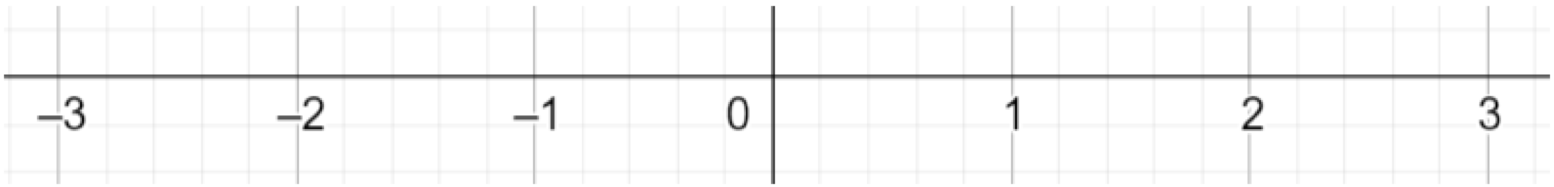
Numbers (2/5)

$\mathbf{N} = \{0, 1, 2, 3, \ldots\}$, the set of all **natural numbers**



Numbers (3/5)

$\mathbf{N} = \{0, 1, 2, 3, \ldots\}$, the set of all **natural numbers**
$\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of all **integers**
$\mathbf{Z}^+ = \{1, 2, 3, \ldots\}$, the set of all **positive integers**



Numbers (4/5)

$\mathbf{N} = \{0, 1, 2, 3, \ldots\}$, the set of all **natural numbers**

$\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of all **integers**

$\mathbf{Z}^+ = \{1, 2, 3, \ldots\}$, the set of all **positive integers**

$\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, \text{ and } q \neq 0\}$,
the set of all **rational numbers**

Numbers (5/5)

$\mathbf{N} = \{0, 1, 2, 3, \ldots\}$, the set of all **natural numbers**

$\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, the set of all **integers**

$\mathbf{Z}^+ = \{1, 2, 3, \ldots\}$, the set of all **positive integers**

$\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, \text{and } q \neq 0\}$,
the set of all **rational numbers**

$\mathbf{R}$, the set of all **real numbers**

$\mathbf{R}^+$, the set of all **positive real numbers**

Sets (1/11)

A **set** is an unordered collection of objects.

The objects in a set are called the *elements*, or *members*, of the set. A set is said to contain its elements.

Sets (2/11)

$S = \{a, b, c, d\}$

We write $a \in S$ to denote that a is an element of the set S. The notation $e \notin S$ denotes that e is not an element of the set S.

Sets (3/11)

Example1:

For each of the following sets,
determine whether 3 is an element of that set.

$\{1,2,3,4\}$

$\{\{1\},\{2\},\{3\},\{4\}\}$

$\{1,2,\{1,3\}\}$

Sets (3/11)

Example1:

For each of the following sets,
determine whether 3 is an element of that set.

$3 \in \{1,2,3,4\}$

$3 \notin \{\{1\},\{2\},\{3\},\{4\}\}$

$3 \notin \{1,2,\{1,3\}\}$

Empty Set

There is a special set that has no elements. This set is called the empty set, or null set, and is denoted by $\emptyset$.

The empty set can also be denoted by { }

Sets (5/11)

Cardinality

The cardinality is the number of distinct elements in S. The cardinality of S is denoted by $|S|$.

Sets (6/11)

Example1

$$S = \{a, b, c, d\}$$
$$|S| = 4$$

$$A = \{1, 2, 3, 7, 9\}$$
$$|A| = 5$$

$$\emptyset = \{\ \}$$
$$|\emptyset| = 0$$

Sets (7/11)

Example2

$S = \{a, b, c, d, \{2\}\}$
$|S| = 5$

$A = \{1, 2, 3, \{2,3\}, 9\}$
$|A| = 5$

$\{\emptyset\} = \{\{\ \}\}$
$|\{\emptyset\}| = 1$

Sets (8/11)

Infinite

A set is said to be **infinite** if it is not finite.
The set of positive integers is infinite.

$$Z^+ = \{1,2,3, \ldots\}$$

Sets (9/11)

If A and B are sets, then A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$. We write $A = B$, if A and B are equal sets.

- The sets $\{1, 3, 5\}$ and $\{3, 5, 1\}$ are equal, because they have the same elements.
- $\{1, 3, 3, 5, 5, 5\}$ is the same as the set $\{1, 3, 5\}$ because they have the same elements.

Subset

The set A is said to be a subset of B if and only if every element of A is also an element of B .

We use the notation $A \subseteq B$ to indicate that A is a subset of the set B .

$$A \subseteq B \leftrightarrow \forall x(x \in A \rightarrow x \in B)$$

Subset

For every set S,
$$(i)\ \emptyset \subseteq S \quad \text{and} \quad (ii)\ S \subseteq S.$$

To show that two sets A and B are equal, show that $A \subseteq B$ and $B \subseteq A$.

Proper Subset

The set A is a subset of the set B but that $A \neq B$, we write $A \subset B$ and say that A is a **proper subset** of B.

$$A \subset B \leftrightarrow \big(\forall x(x \in A \rightarrow x \in B) \land \exists x(x \in B \land x \notin A)\big)$$

Functions (1/12)

The Function:

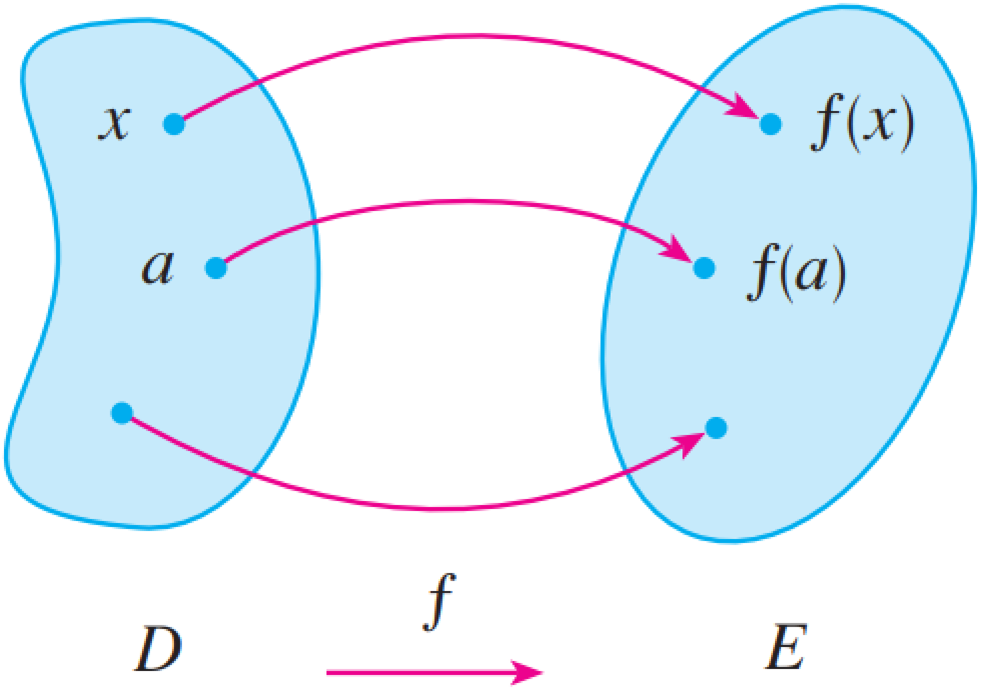
Machine

Ex. $\sqrt{}$

$4 \rightarrow 2$
$16 \rightarrow 4$
...

Functions (2/12)

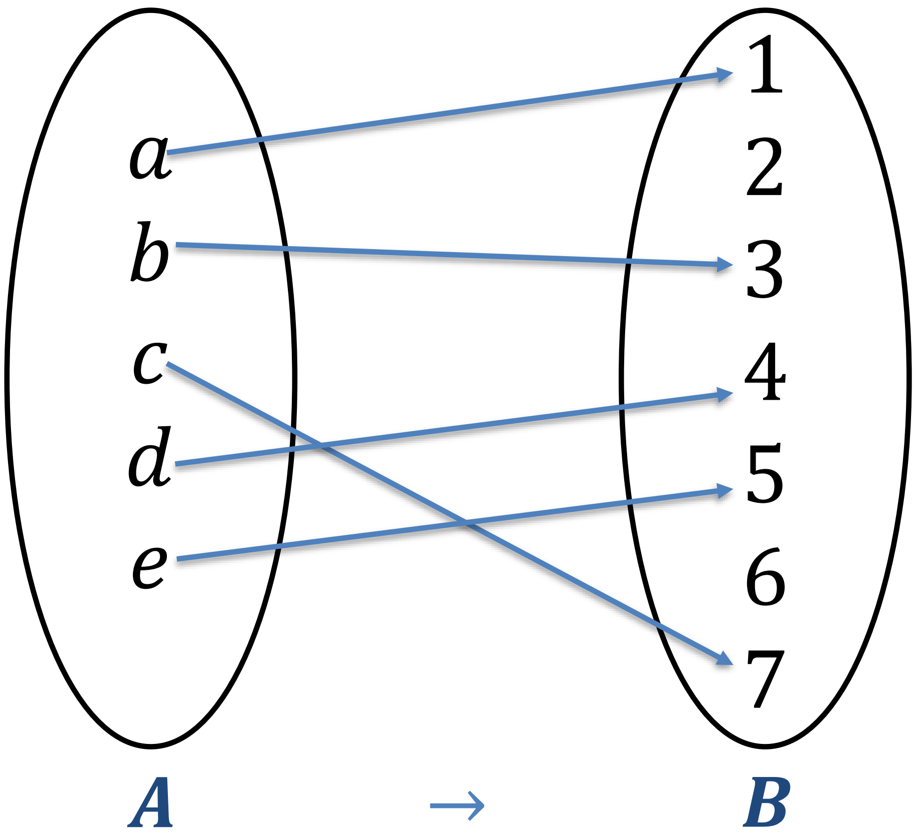
The Function $f: D \to E$



A **function** f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.

Functions (3/12)

The Function $f: A \to B$



$\text{Domain} = \{a, b, c, d, e\}$

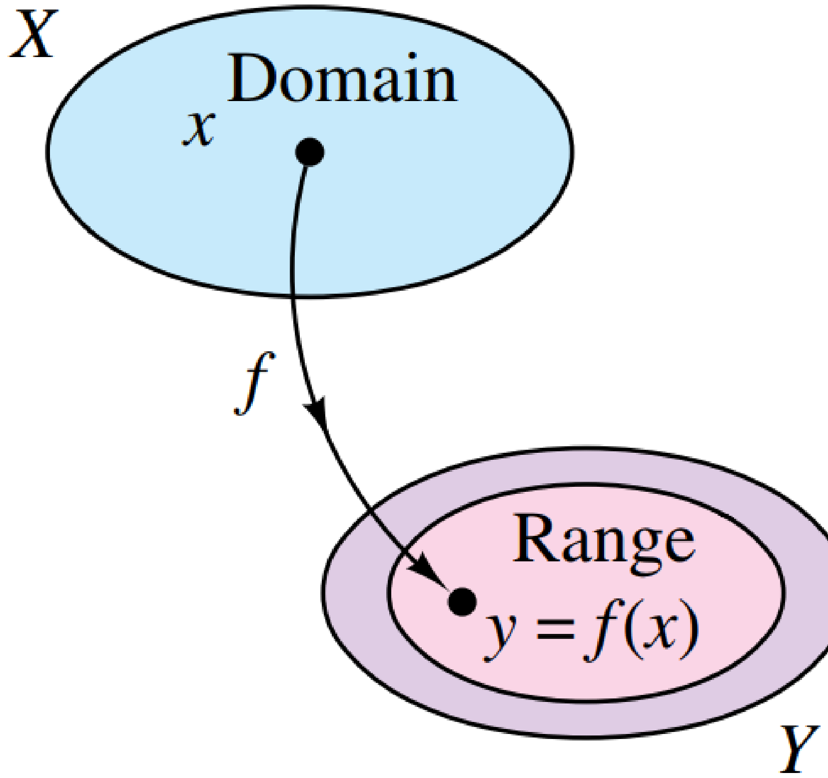
$\text{Co-Domain} = \{1,2,3,4,5,6,7\}$

$\text{Range} = \{1,3,4,5,7\}$

ex. $f(a) = 1,\ f(e) = 5, \ldots$

Functions (4/12)

The Function $f: X \to Y$

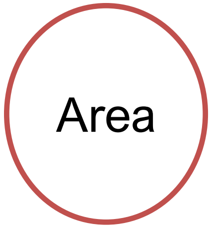


Functions (5/12)

Function of Circle's Area:

- The area A of a circle depends on the radius r of the circle. The rule that connects r and A is given by the equation $a = \pi r^2$. With each positive number r there is associated one value of A, and we say that A is a *function* of r.

- $A = f(r) = \pi r^2$

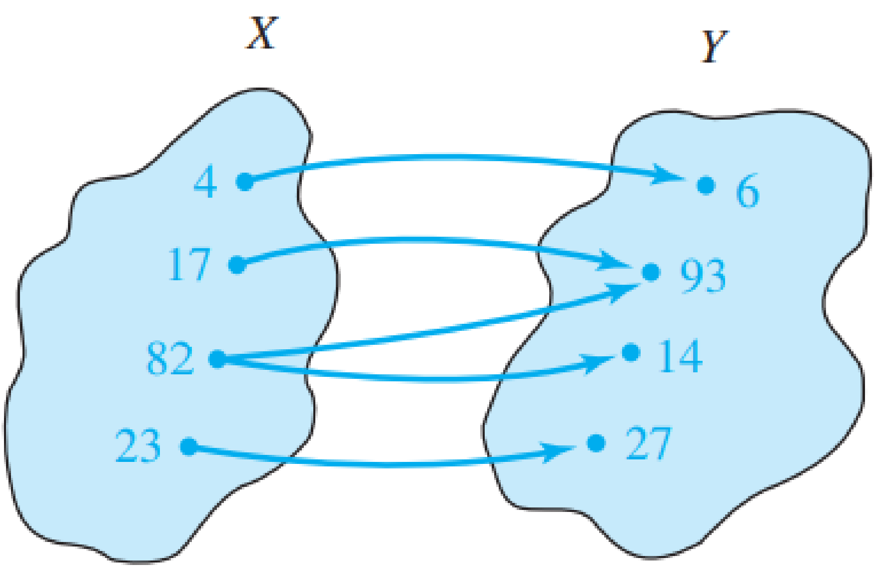


where r is the **independent** variable and A is the **dependent** variable.

Functions (6/12)

Example4:

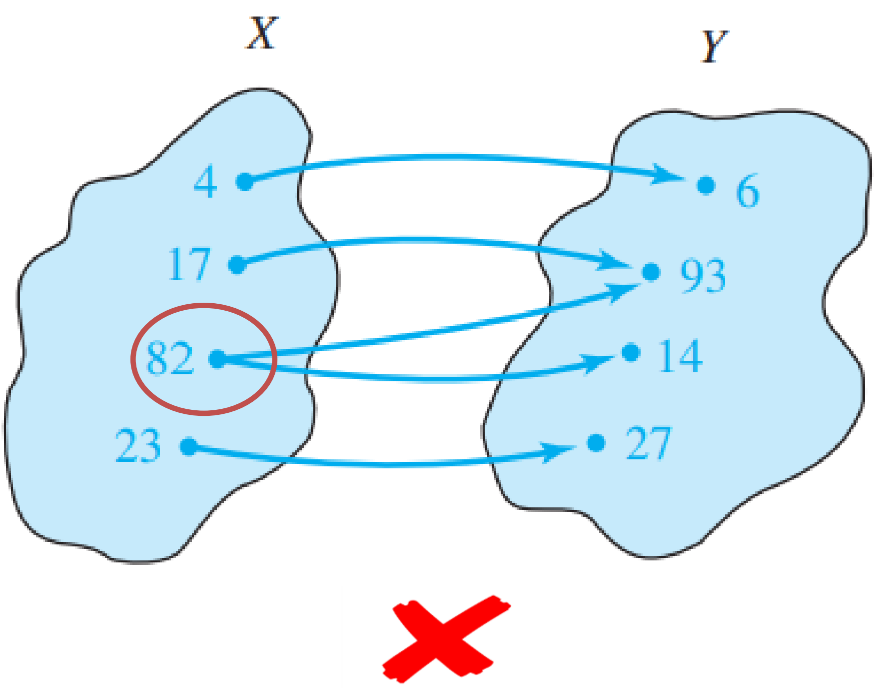
Is the following rules define y as a function of x?



Functions (6/12)

Example4:

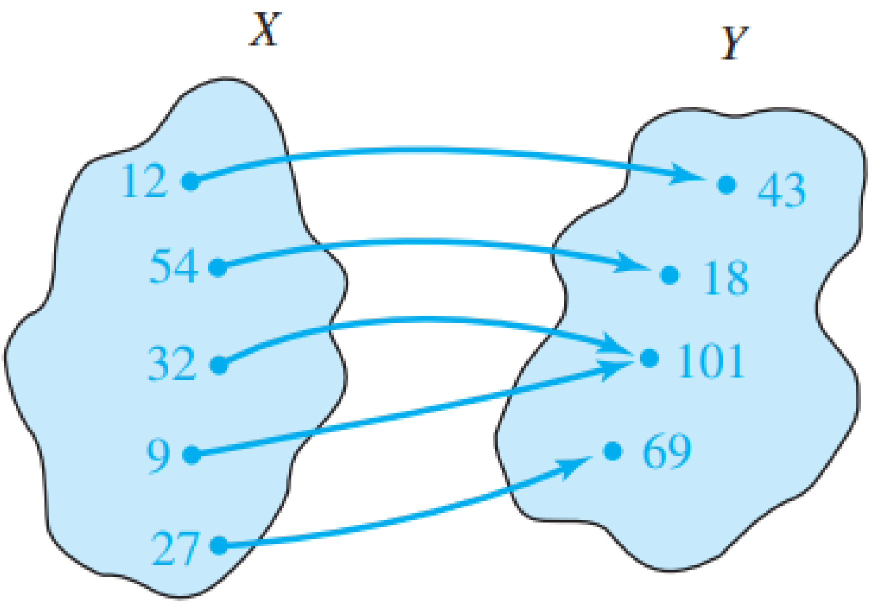
Is the following rules define y as a function of x?



Functions (7/12)

Example5:

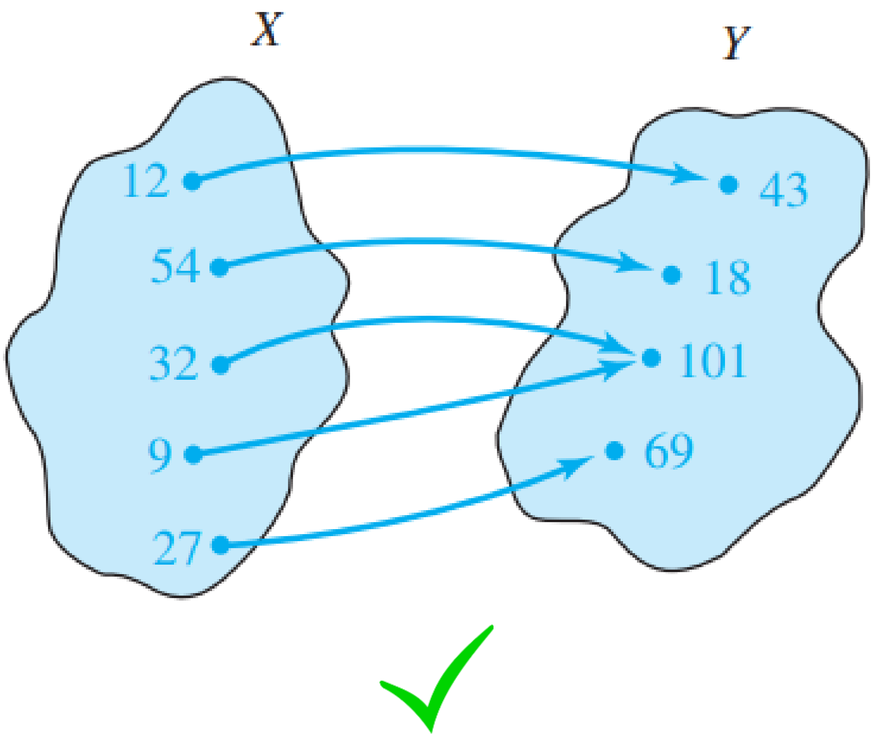
Is the following rules define y as a function of x?



Functions (7/12)

Example5:

Is the following rules define y as a function of x?



Functions (8/12)

Representations of Functions:

- The function can be represented in different ways:
  - by an equation,
  - in a table,
  - in words,
  - Or by a graph.

Functions (9/12)

Representations of Functions (Equation):

- Equation in implicit form:

$$x^2 + 2y = 1$$

- Equation in explicit form:

$$y = \frac{1}{2}(1 - x^2)$$

- Function notation

$$f(x) = \frac{1}{2}(1 - x^2)$$

Representations of Functions (Table):

$$f(x) = 2x$$

x	$f(x)$
-2	-4
-1	-2
0	0
1	2
2	4
0.5	1

Functions (11/12)

Representations of Functions (in words (verbally)):

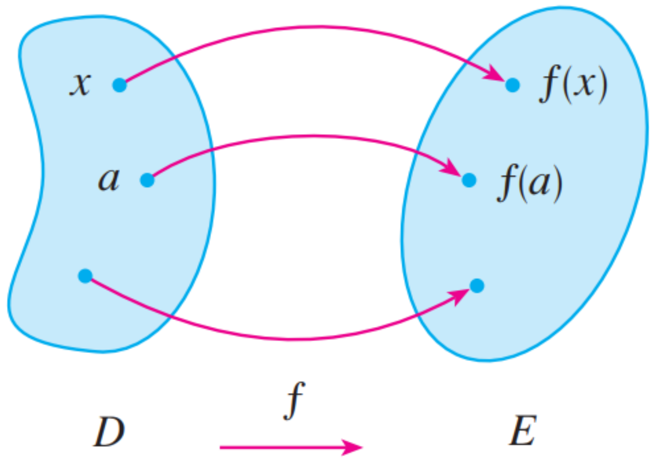
$$f(x) = x^2$$

- The function is described in words:

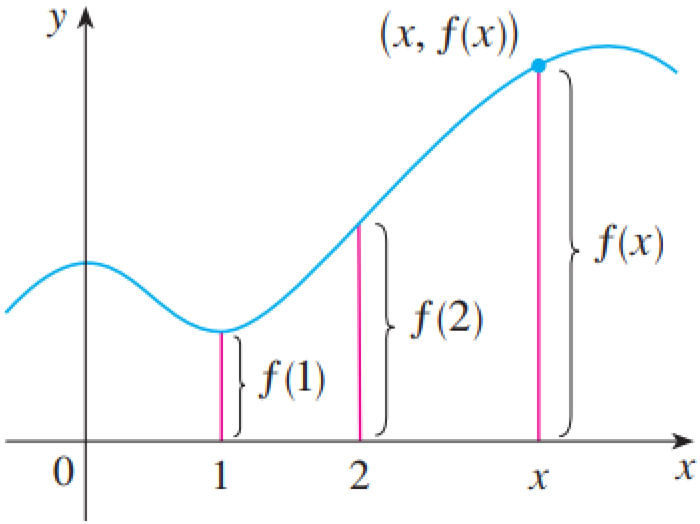
Let $f(x)$ be the squared value of x. The rule that the value of squared x is equal to $x \times x$.

Functions (12/12)

Representations of Functions (Graph):



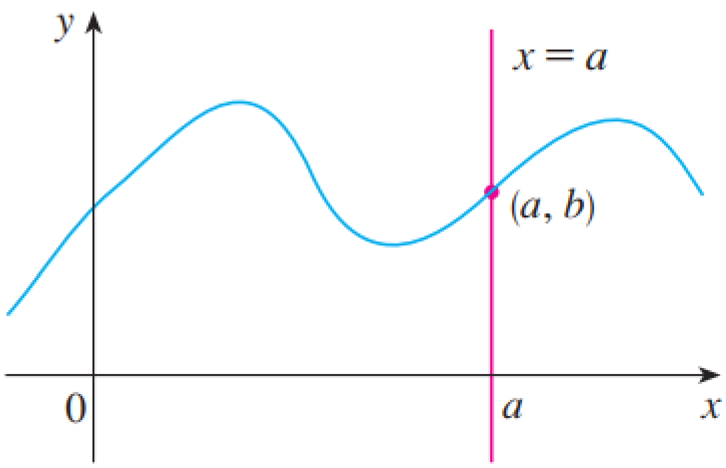
Arrow Diagram



Graph

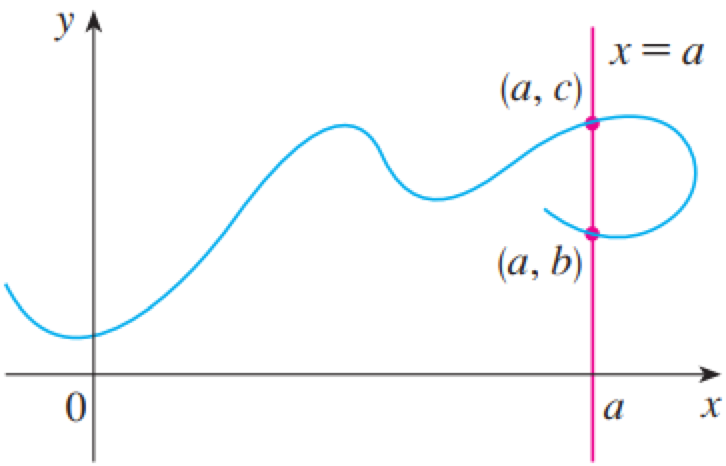
Representations of Functions (Graph):

The Vertical Line Test:



(a) This curve represents a function.

$$y = f(x)$$



(b) This curve doesn't represent a function.

$$y = f(x)$$

Domain & Range (1/37)

Four types of **inequalities** to compare real numbers:

$x < y$	x is less than y
$x \le y$	x is less than or equal to y
$x > y$	x is greater than y
$x \ge y$	x is greater than or equal to y

The double inequality $a < b < c$ is shorthand for the pair of inequalities $a < b$ and $b < c$.

Domain & Range (2/37)

Intervals on the Number Line

1

Inequality

$a \le x \le b$

Geometric Description

Interval Notation

$[a, b]$

Closed Interval

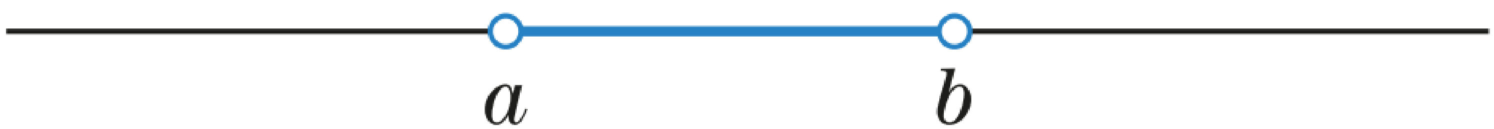
Domain & Range (2/37)

Intervals on the Number Line

2

Inequality
$a < x < b$

Geometric Description



Interval Notation
(a, b)

Opened Interval

Domain & Range (2/37)

Intervals on the Number Line

3

Inequality
$a \le x < b$

Geometric Description

Interval Notation
$[a, b)$

Domain & Range (2/37)

Intervals on the Number Line

4

Inequality

$$a < x \le b$$

Geometric Description

Interval Notation

$$(a, b]$$

Domain & Range (2/37)

Intervals on the Number Line

5

Inequality

$a \leq x$

Geometric Description

Interval Notation

$[a, \infty)$

Domain & Range (2/37)

Intervals on the Number Line

6

Inequality
$a < x$

Geometric Description

Interval Notation
(a, ∞)

Domain & Range (2/37)

Intervals on the Number Line

7

Inequality

$x \leq b$

Geometric Description

Interval Notation

$(-\infty, b]$

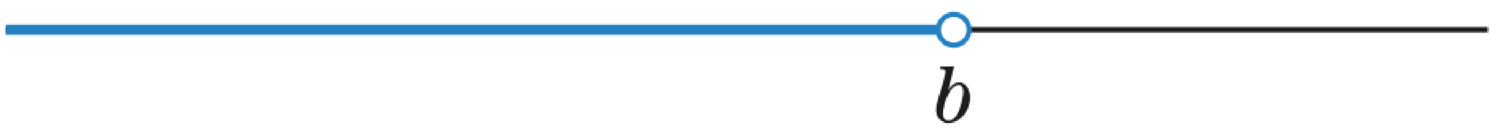
Domain & Range (2/37)

Intervals on the Number Line

8

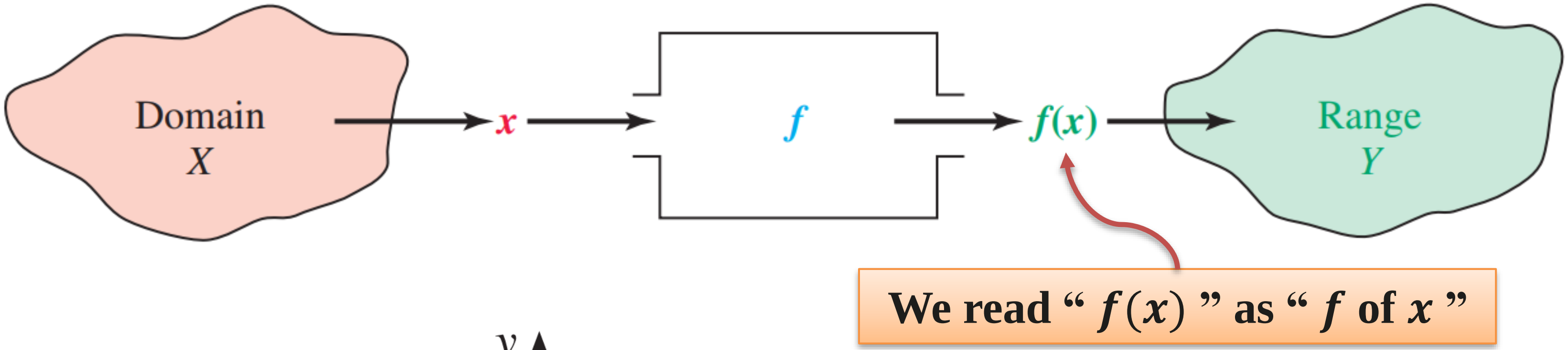
Inequality
$x < b$

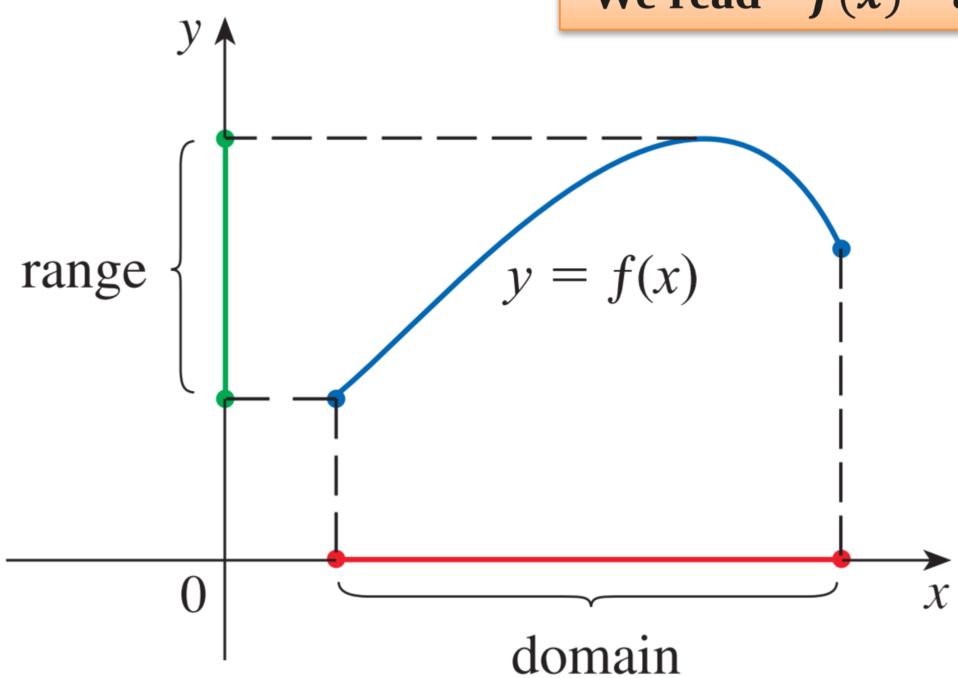
Geometric Description



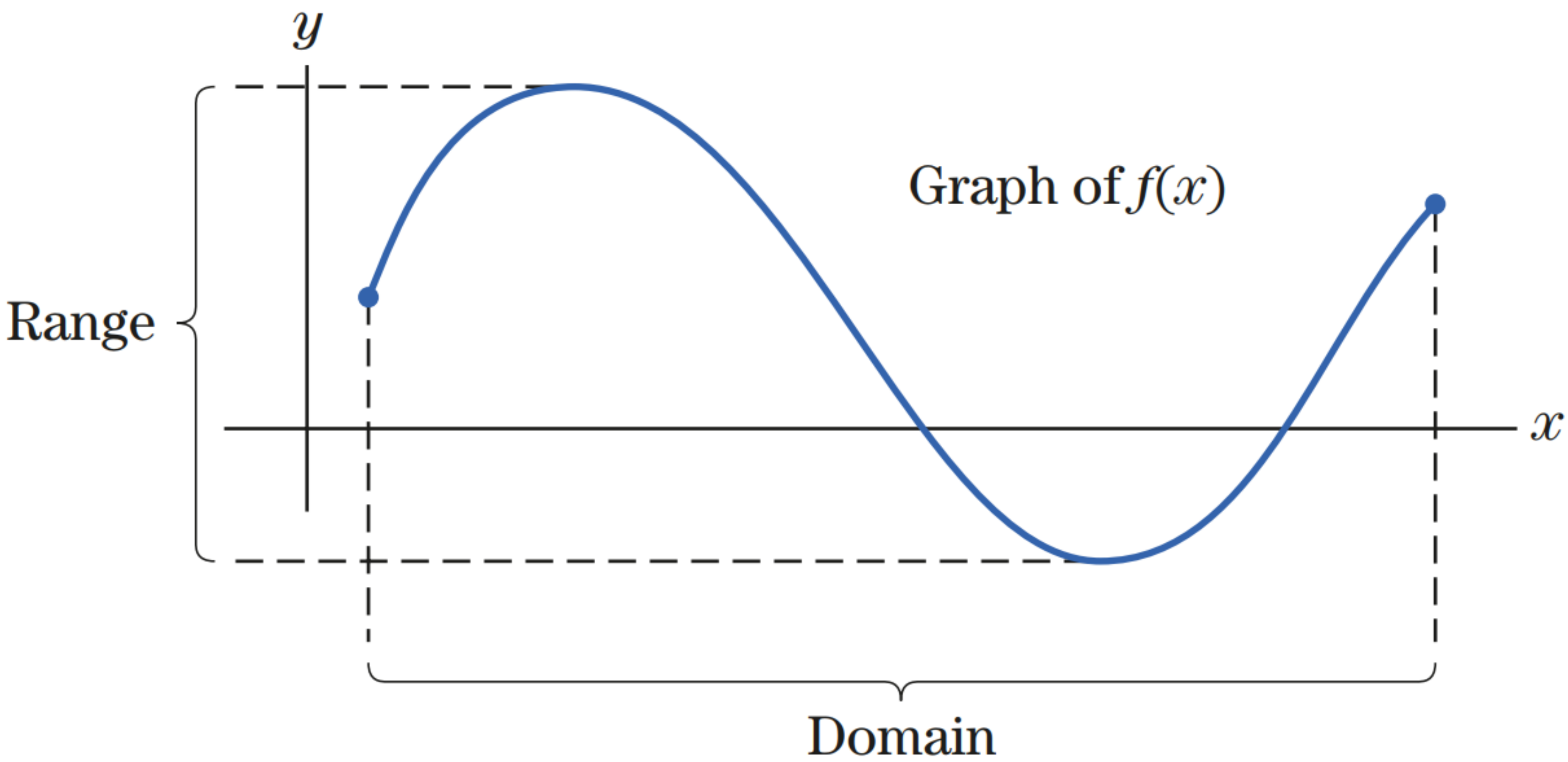
Interval Notation
$(-\infty, b)$

Domain & Range (13/37)





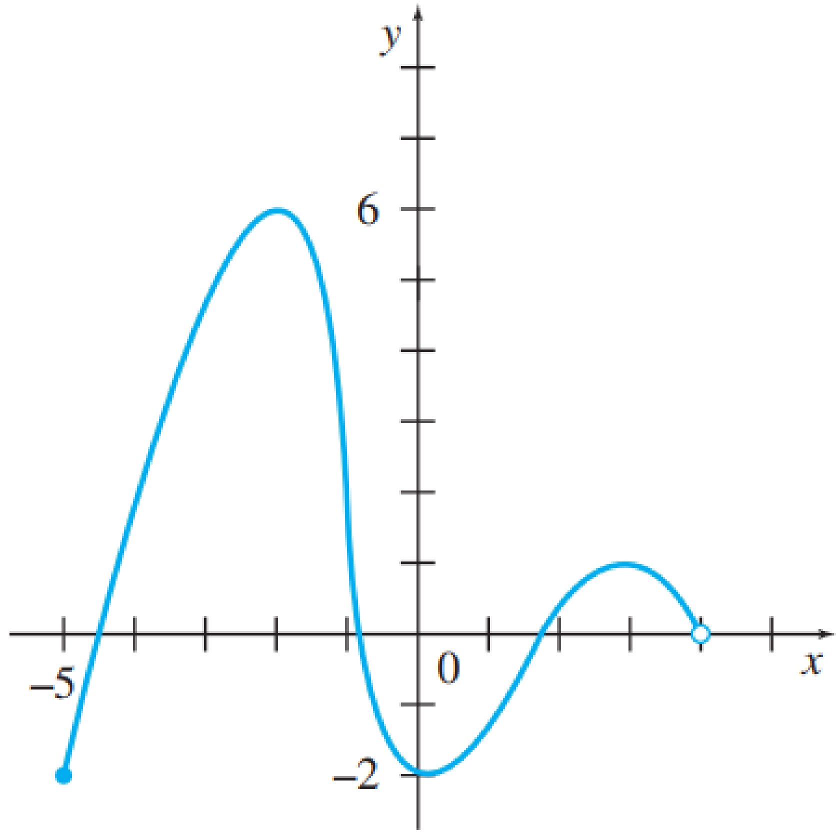
Domain & Range (14/37)



Domain & Range (15/37)

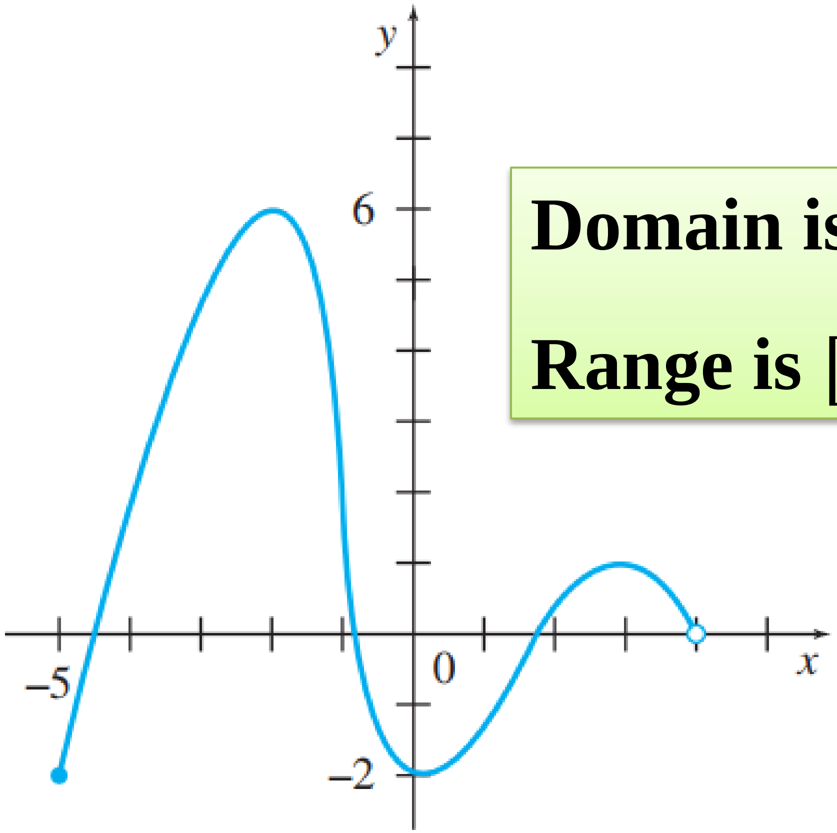
Example1:

Find the domain and range of the following function.



Example1:

Find the domain and range of the following function.



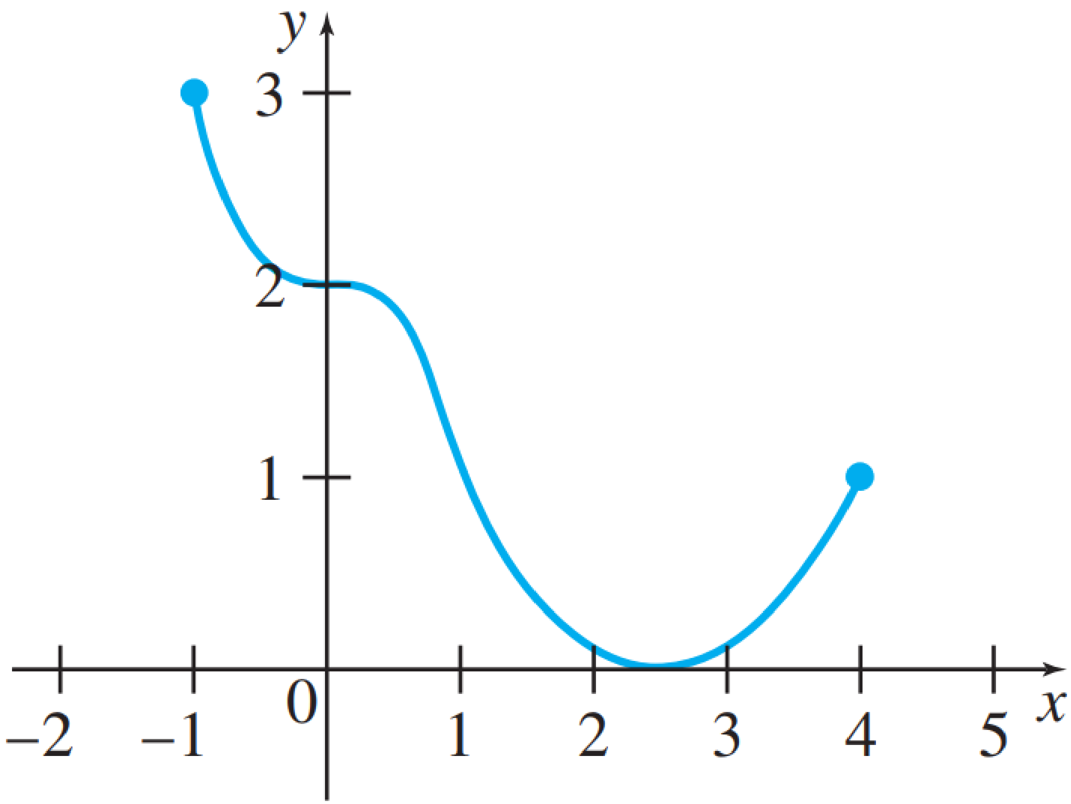
Domain is $[-5, 4)$.

Range is $[-2, 6]$.

Domain & Range (16/37)

Example2:

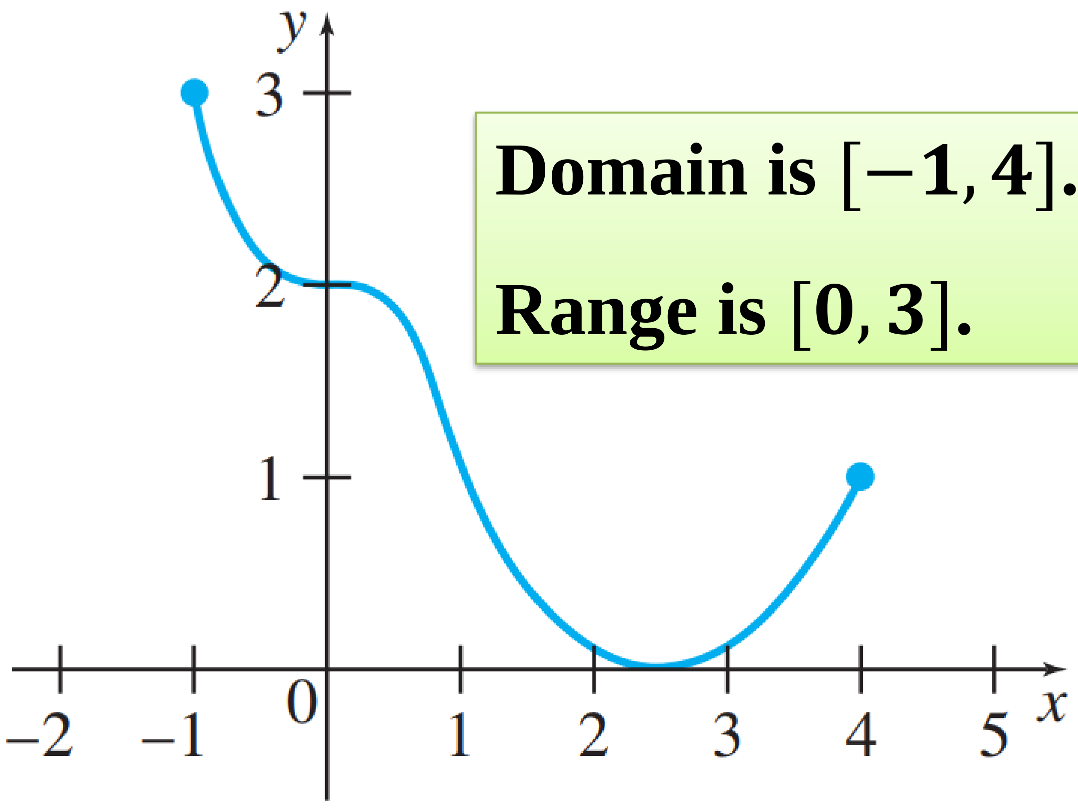
Find the domain and range of the following function.



Domain & Range (16/37)

Example2:

Find the domain and range of the following function.

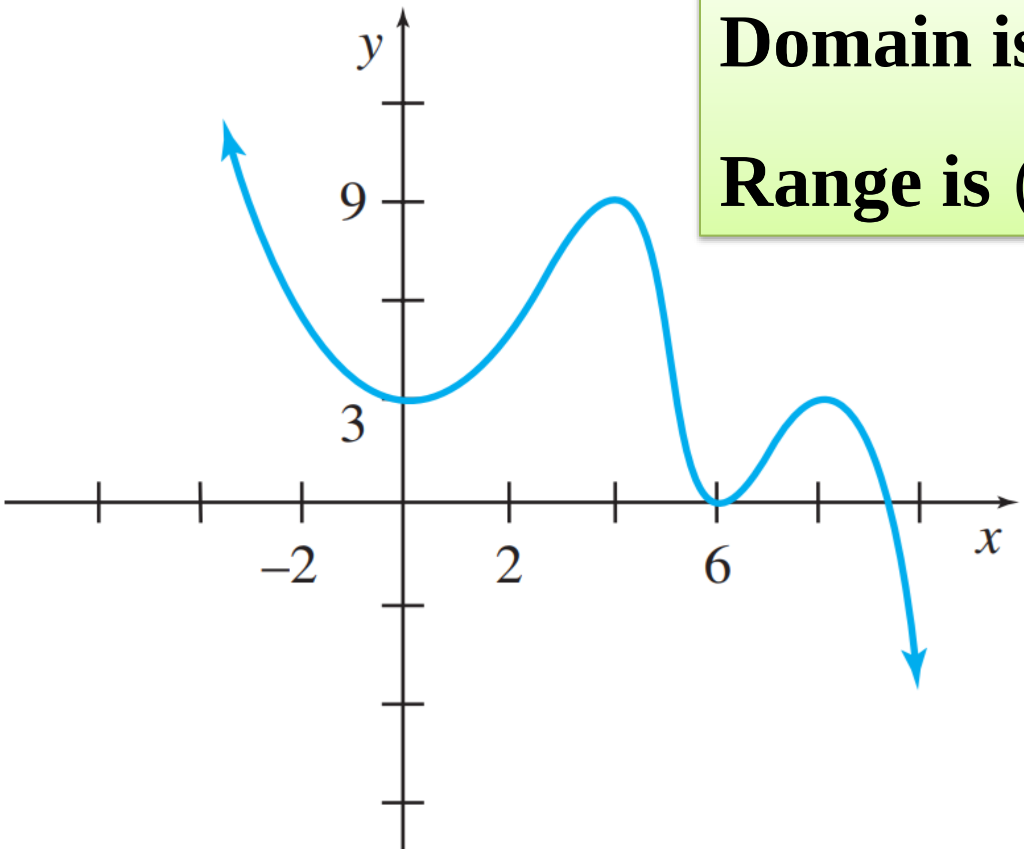


Domain is $[-1, 4]$.

Range is $[0, 3]$.

Example3:

Find the domain and range of the following function.



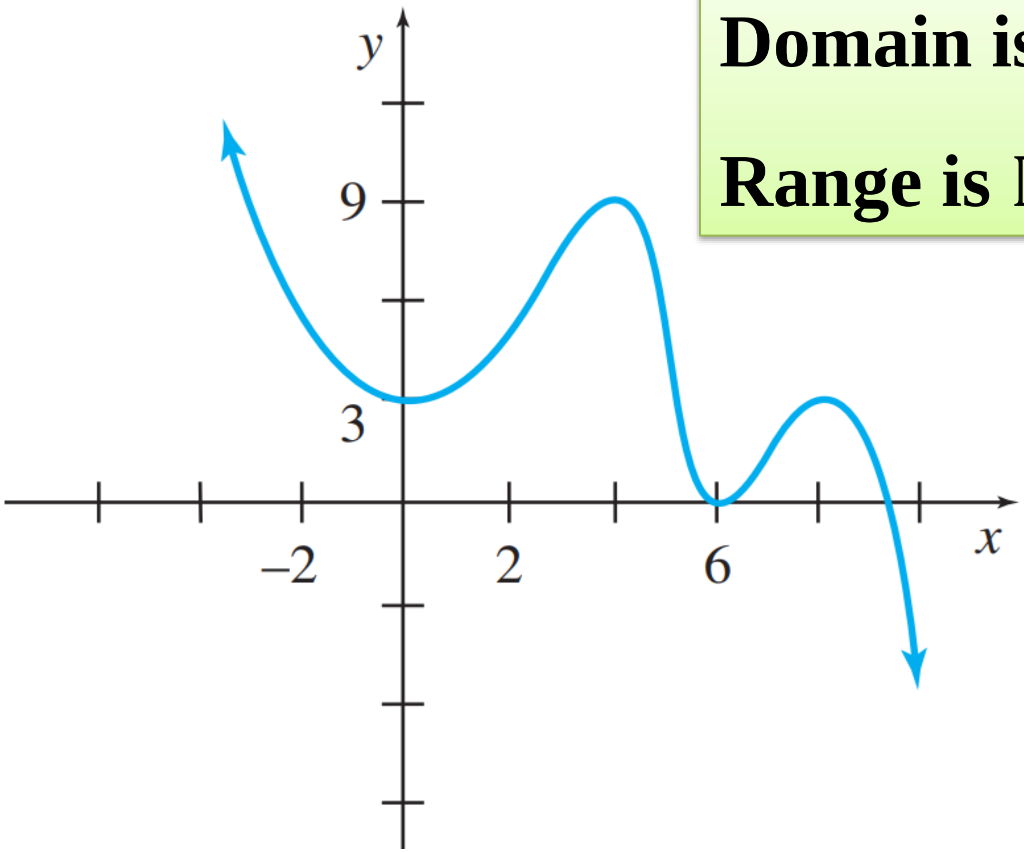
Domain is $(-\infty, \infty)$.

Range is $(-\infty, \infty)$.

Domain & Range (17/37)

Example3:

Find the domain and range of the following function.



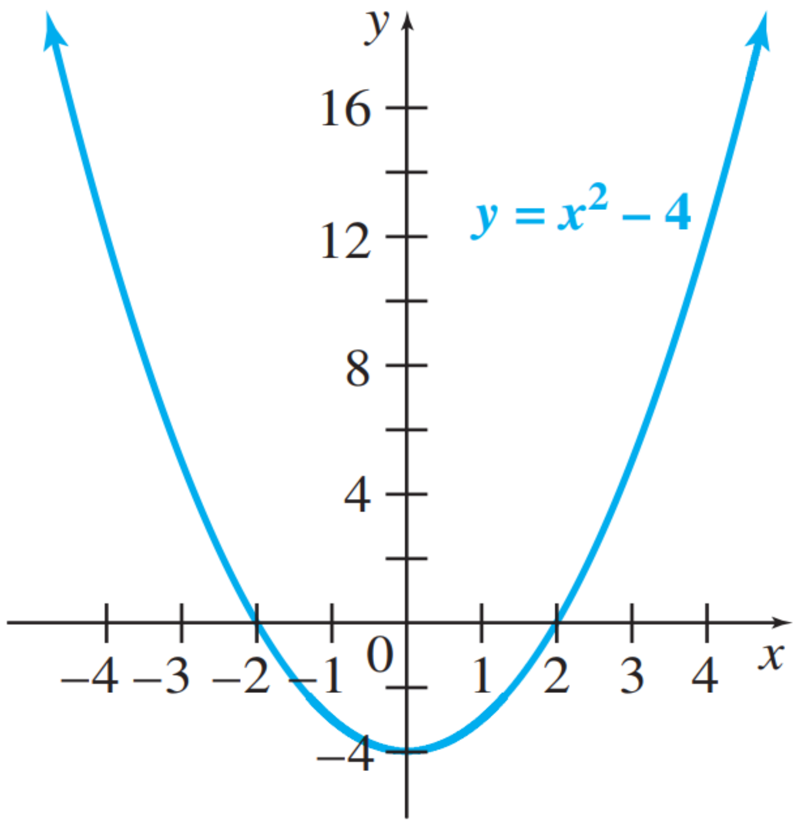
Domain is $\mathbb{R}$.

Range is $\mathbb{R}$.

Domain & Range (18/37)

Example4:

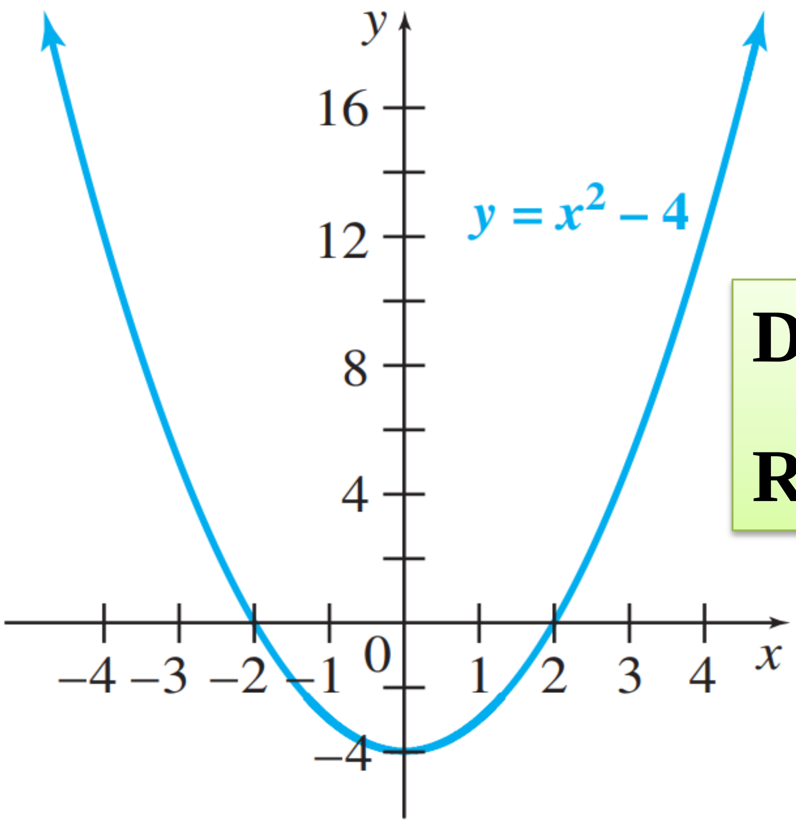
Find the domain and range of the following function.



Domain & Range (18/37)

Example4:

Find the domain and range of the following function.



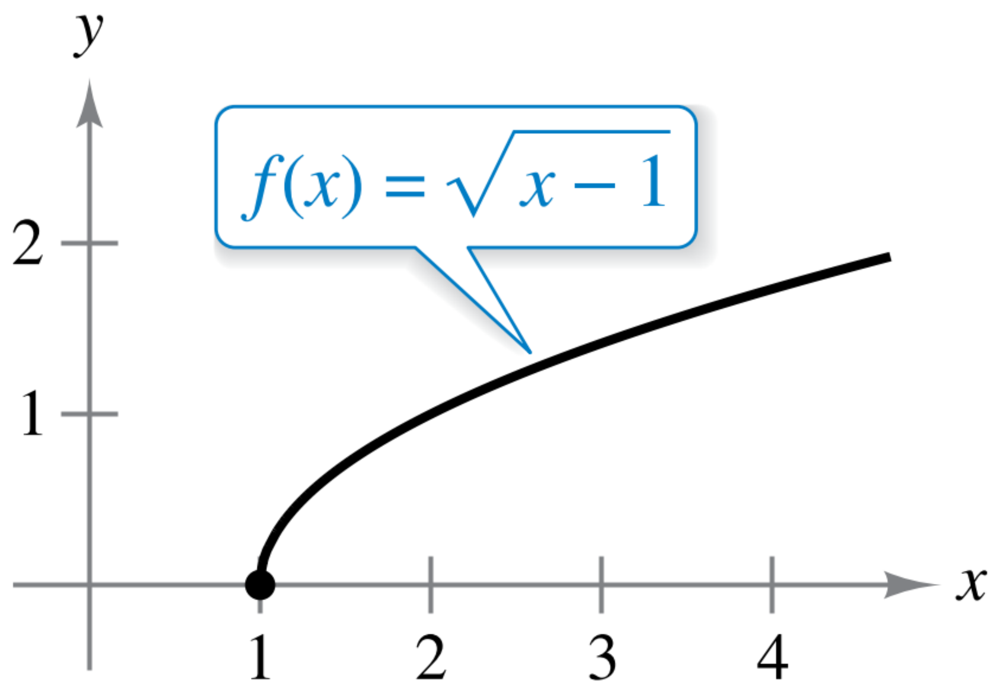
Domain is $\mathbb{R}$.

Range is $[-4, \infty)$.

Domain & Range (19/37)

Example5:

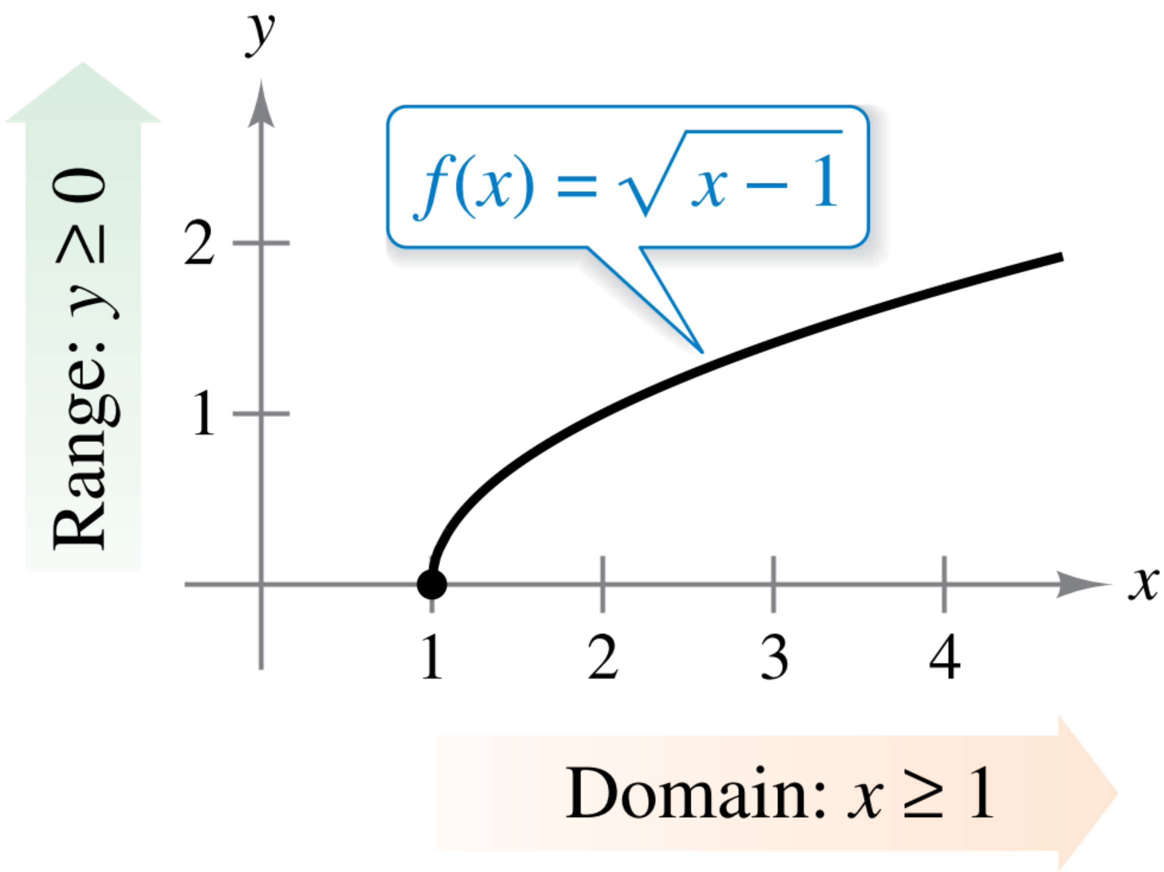
Find the domain and range of the following function.



Domain & Range (19/37)

Example5:

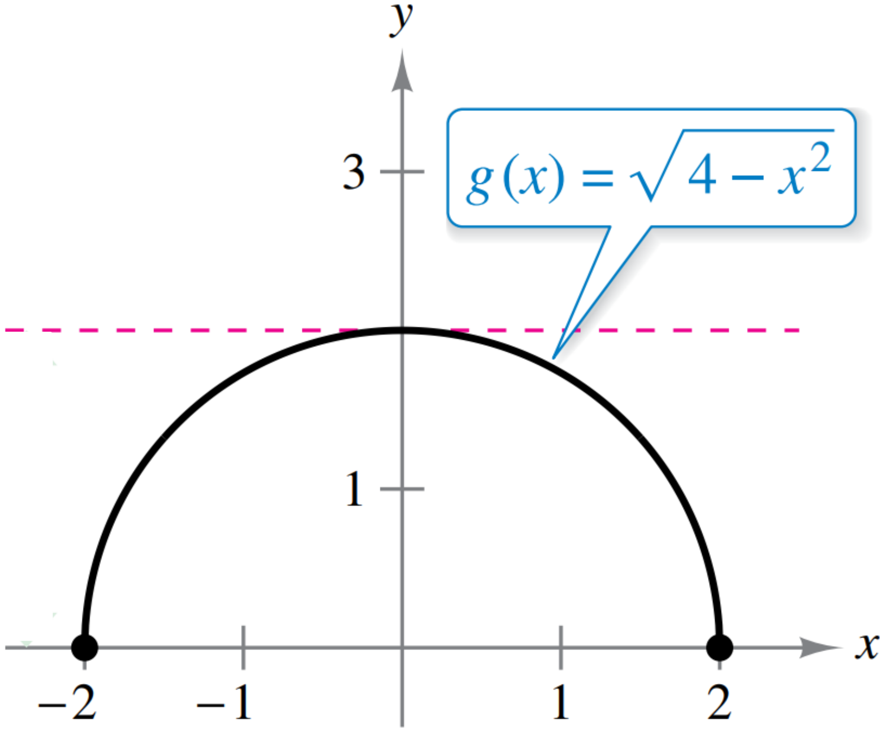
Find the domain and range of the following function.



Domain & Range (20/37)

Example6:

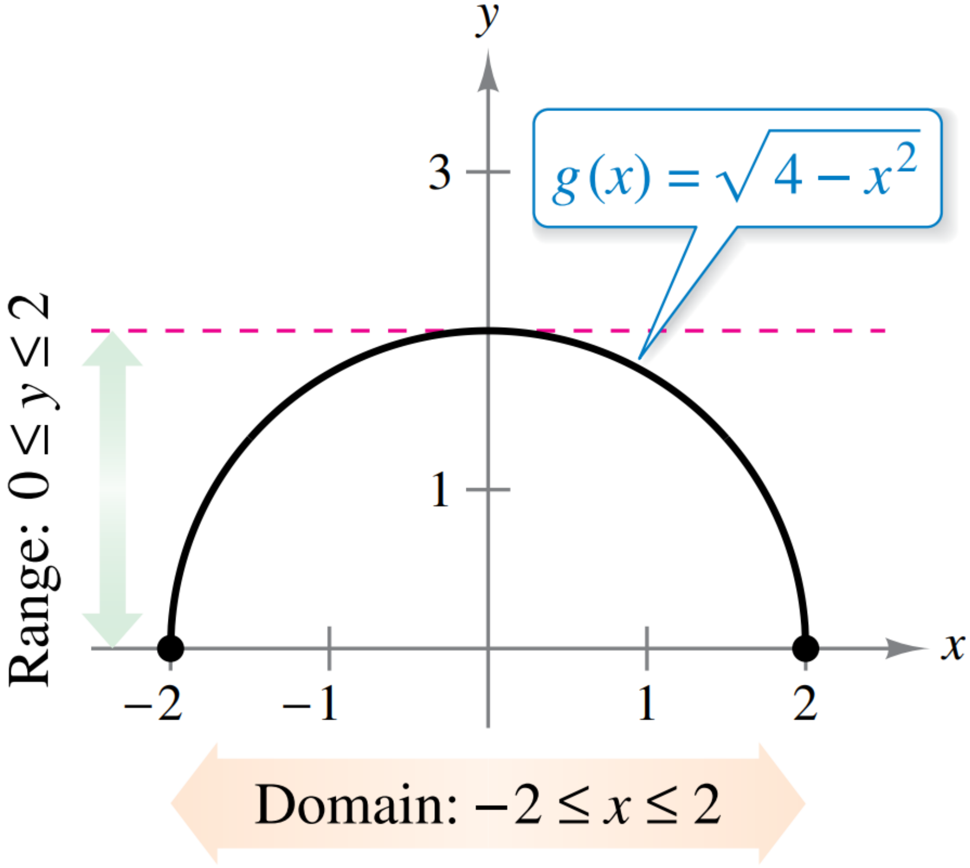
Find the domain and range of the following function.



Domain & Range (20/37)

Example6:

Find the domain and range of the following function.



Implied Domain:

- The domain of a function can be described *explicitly*, or it may be described *implicitly* by an equation used to define the function. The **implied domain** is the set of all real numbers for which the equation is defined, whereas an explicitly defined domain is one that is given along with the function.

Domain & Range (22/37)

For example, on one hand, the function

$$f(x) = x + 2, \qquad 1 \leq x \leq 4$$

has an explicitly defined domain given by $\{x: 1 \leq x \leq 4\}$.

On the other hand, the function

$$f(x) = x + 2$$

has an implied domain … ?

Domain & Range (22/37)

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$$f(x) = x + 2, \qquad 1 \leq x \leq 4$$

has an explicitly defined domain given by $\{x: 1 \leq x \leq 4\}$.

On the other hand, the function

$$f(x) = x + 2 \qquad (-\infty, \infty)$$

has an implied domain that is the set of real numbers.

Domain & Range (23/37)

Polynomial Function:

The most common type of algebraic function is a polynomial function: A function P is called a polynomial if

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots, a_n$ are constants called the coefficients of the polynomial. **The domain of any polynomial is $\mathbb{R}$.**

Domain & Range (24/37)

Example1:

Find the domain of the following function.

$$f(x) = x^2$$

Any number may be squared, so the domain is the set of all real numbers $\mathbb{R}$, written $(-\infty, \infty)$.

Domain & Range (25/37)

Another example, on one hand, the function

$$f(x) = \frac{1}{x^2 - 4}, \quad 4 \leq x \leq 5$$

has an explicitly defined domain given by $\{x\colon 4 \leq x \leq 5\}$.

On the other hand, the function

$$g(x) = \frac{1}{x^2 - 4}$$

has an implied domain ...?

Domain & Range (25/37)

Another example, on one hand, the function

$$f(x) = \frac{1}{x^2 - 4}, \quad 4 \leq x \leq 5$$

has an explicitly defined domain given by $\{x: 4 \leq x \leq 5\}$.

On the other hand, the function

$$g(x) = \frac{1}{x^2 - 4}$$

$\mathbb{R} - \{\pm 2\}$

has an implied domain that is the set $\{x: x \neq \pm 2\}$.

Domain & Range (26/37)

CAUTION:

When finding the domain of a function, there are two operations to avoid:

(1) dividing by zero; and

(2) taking the square root (or any even root) of a negative number.

Example2:

Find the domain of the following function.

$$f(x) = \frac{3}{x}$$

Example2:

Find the domain of the following function.

$$f(x) = \frac{3}{x}$$

f is defined for $x \neq 0$.

So, the domain of the function is $\mathbb{R} - \{0\}$.

Also, the domain is $(-\infty, 0) \cup (0, \infty)$.

Domain & Range (28/37)

Example3:

Find the domain of the following function.

$$f(x) = \frac{x}{x^2 - 1}$$

Example3:

Find the domain of the following function.

$$f(x) = \frac{x}{x^2 - 1}$$

f is defined for $x^2 - 1 \neq 0$. Therefore, $x \neq \pm 1$.

So, the domain of the function is $\mathbb{R} - \{\pm 1\}$.

Also, the domain is $(-\infty, -1) \cup (-1,1) \cup (1, \infty)$.

Example4:

Find the domain of the following function.

$$f(x) = \frac{2x + 1}{x^2 - x - 12}$$

Example4:

Find the domain of the following function.

$$f(x) = \frac{2x + 1}{x^2 - x - 12}$$

at most two roots

Using Quadratic Formula, the solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example4:

Find the domain of the following function.

$$f(x) = \frac{2x + 1}{x^2 - x - 12}$$

$$x^2 - x - 12 = 0$$

$a = 1$

$b = -1$

$c = -12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example4:

Find the domain of the following function.

$$f(x) = \frac{2x + 1}{x^2 - x - 12}$$

$$x^2 - x - 12 = 0$$

$a = 1$

$b = -1$

$c = -12$

$$x = \frac{1 \pm \sqrt{1 + 48}}{2} = \frac{1 \pm 7}{2}$$

$$x = -3, \qquad x = 4$$

Example4:

Find the domain of the following function.

$$f(x) = \frac{2x+1}{x^2 - x - 12}$$

So, the domain of the function is $\mathbb{R} - \{-3, 4\}$.

Also, the domain is $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$.

Domain & Range (30/37)

Example5:

If

$$f(x) = \frac{x^2 - x}{x - 1} \qquad \text{and} \qquad g(x) = x$$

is it true that $f = g$?

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If

$$f(x) = \frac{x^2 - x}{x - 1} \qquad \text{and} \qquad g(x) = x$$

is it true that $f = g$?

<u>NO</u>, because the domain of the function f is $\mathbb{R} - \{1\}$. However, the domain of the function g is $\mathbb{R}$.

Example6:

Find the domain of the following function.

$$f(x) = \sqrt{x-1}$$

Example6:

Find the domain of the following function.

$$f(x) = \sqrt{x - 1}$$

The domain is the set of all x-values for which $x - 1 \geq 0$, which is the interval $[1, \infty)$.

Example7:

Find the domain of the following function.

$$f(x) = \sqrt{x^2 - x - 6}$$

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Find the domain of the following function.

$$f(x) = \sqrt{x^2 - x - 6}$$

$x^2 - x - 6 \geq 0$

Example7:

Find the domain of the following function.

$$f(x) = \sqrt{x^2 - x - 6}$$

$$x^2 - x - 6 = 0$$

$a = 1$

$b = -1$

$c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example7:

Find the domain of the following function.

$$f(x) = \sqrt{x^2 - x - 6}$$

$$x^2 - x - 6 = 0$$

$a = 1$

$b = -1$

$c = -6$

$$x = \frac{1 \pm \sqrt{1 + 24}}{2} = \frac{1 \pm 5}{2}$$

$$x = -2 \quad , \quad x = 3$$

Domain & Range (32/37)

Example7:

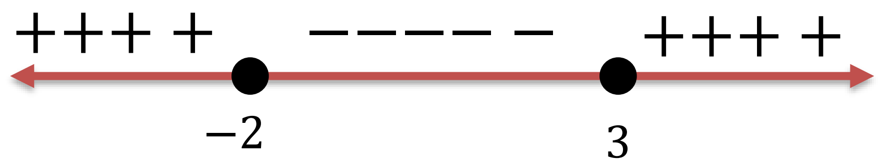
Find the domain of the following function.

$$f(x) = \sqrt{x^2 - x - 6}$$

$$x^2 - x - 6 \geq 0$$

$$(x+2)(x-3) \geq 0$$

Roots: $x = -2$, $x = 3$



Example7:

Find the domain of the following function.

$$f(x) = \sqrt{x^2 - x - 6}$$

$$x^2 - x - 6 \geq 0$$

$$(x+2)(x-3) \geq 0$$

Roots: $x = -2$, $x = 3$

Therefore, the domain is $(-\infty, -2] \cup [3, \infty)$.

Domain & Range (33/37)

Example9:

Find the domain of the following function.

$$f(x) = \frac{2x + 3}{\sqrt{x - 2}}$$

Example9:

Find the domain of the following function.

$$f(x) = \frac{2x + 3}{\sqrt{x - 2}}$$

Numerator is defined for all real numbers.

Denominator is defined for $x - 2 > 0$. Then, $x > 2$.

So, the domain of the function is the interval $(2, \infty)$.

Domain & Range (34/37)

Example10:

Find the domain of the following function.

$$f(x) = \frac{2x + 3}{\sqrt{x} - 2}$$

Example10:

Find the domain of the following function.

$$f(x) = \frac{2x + 3}{\sqrt{x} - 2}$$

Numerator is defined for all real numbers.

Denominator is defined for:

1) $x \geq 0$ and 2) $\sqrt{x} - 2 \neq 0$ (i.e., $x \neq 4$)

So, the domain of the function is the interval $[0, \infty) - \{4\}$.

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6hMiR2Xis-mJ1sXNwWsZ1Bh

Lecture #1: https://www.youtube.com/watch?v=pUAaasolcVk&list=PLxIvc-MGOs6hMiR2Xis-mJ1sXNwWsZ1Bh&index=1

https://www.youtube.com/watch?v=DWc2gOSddpM&list=PLxIvc-MGOs6hMiR2Xis-mJ1sXNwWsZ1Bh&index=2

https://www.youtube.com/watch?v=I7T6DBeMaOA&list=PLxIvc-MGOs6hMiR2Xis-mJ1sXNwWsZ1Bh&index=3

https://www.youtube.com/watch?v=O9wg9Pgpa6c&list=PLxIvc-MGOs6gkSI_PPAVJpebKDLo-ijEC&index=2

Up to time 01:41:35

Thank You

Dr. Ahmed Hagag

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